Flash 3.3 Documentation Supplement

Aaron Froese
General Fusion Inc.
108–3680 Bonneville Place
Burnaby, BC, Canada
www.generalfusion.com

Feb 10, 2012

Chapter 1

Magnetohydrodynamics

1.1 Anisotropic MHD

Magnetic, viscous, and heat transport occur at different rates parallel and perpendicular to the field in a strongly magnetized plasma. This anisotropy must be taken into account when evaluating energy containment time in magnetic confinement fusion applications. The classical transport parameters were derived by Braginskii and can be found in the NRL plasma formulary.

The Anisotropic MHD unit is based on the Unsplit Staggered Mesh MHD solver and uses all the same switches, in addition to parameters that control the transport. This unit is not compatible with the existing isotropic transport units for magnetic resistivity, viscosity, and conductivity units. The new switches are as follows. The parameter max_flux is a holdover from a previous method of calculating Δt. It can be ignored.

```
## -------------------------------------------------------##
##       Resistive MHD parameters                         ##

# Transport unit flags
useAnisotropicConductivity = .true.
useAnisotropicResistivity  = .true.
useAnisotropicViscosity = .true.

use_Braginskii             = .true.
#If Braginskii_Z_dependence is false, Z=1 is assumed.
Braginskii_Z_dependence    = .false.

#If use_Braginskii is false, the following parameters act
#as prescribed constant values.  If use_Braginskii is true,
#then each positive value is treated as a maximum.
viscosity_0                = 1.
viscosity_1                = 1.
viscosity_2                = 1.
viscosity_3                = 1.
viscosity_4                = 1.
conductivity_para          = 1.e3
conductivity_perp          = 1.e3
conductivity_caret         = 1.e3
resistivity_para           = 1.e-4
resistivity_perp           = 1.e-4
resistivity_Hall           = 1.e-4

# Constraint on dt, preventing fast changes in density
# and energy due to transport fluxes
max_flux                = 1.e-3

## -------------------------------------------------------##
```

In the Braginskii formulation, the important time scales are the collision frequencies and gyrofrequencies. The electron and ion collision frequencies given in SI units are

$$\begin{aligned}
\nu_e = \tau_e^{-1} &= \frac{ne^4 \ln \Lambda}{6\sqrt{2}\pi^{3/2}\varepsilon_0^2\sqrt{m_e}T_e^{3/2}}. \\
\nu_i = \tau_i^{-1} &= \frac{ne^4 \ln \Lambda}{12\pi^{3/2}\varepsilon_0^2\sqrt{m_i}T_i^{3/2}}.
\end{aligned}$$

where $n = n_e = n_i$ is the number density of the particles and $\ln \Lambda$ is the Coulomb logarithm. The electron and ion gyrofrequencies are

$$\omega_{ce} = eB/m_e, \qquad \omega_{ci} = eB/m_i,$$

and the ratios of these frequencies are defined as the electron and ion skin depths

$$\delta_e = \omega_{ce}/\nu_e, \qquad \delta_i = \omega_{ci}/\nu_i.$$

1.1.1 Heat Conductivity

The electron heat flux due to temperature gradients is given by

$$\mathrm{q}_T^e = -\kappa_{||}^e \nabla_{||}(kT_e) - \kappa_\perp^e \nabla_\perp(kT_e) - \kappa_\wedge^e \mathbf{b} \times \nabla_\perp(kT_e),$$

where $\kappa_{||}$ is the parallel heat conductivy, $\kappa_\perp$ is the perpendicular heat conductivity, $\kappa_\wedge$ is the thermal gradient drift, and $\mathbf{b}$ is the unit vector parallel to the magnetic field. The thermal conductivities are

$$\begin{aligned}
\kappa_{||}^e &= \kappa_{||}^0(\bar{Z})\frac{nkT_e\tau_e}{m_e} \\
\kappa_\perp^e &= \kappa_\perp^0(\bar{Z})\frac{nkT_e\tau_e}{m_e\delta_e^2} \\
\kappa_\wedge^e &= \frac{5}{2}\frac{nkT_e\tau_e}{m_e\delta_e},
\end{aligned}$$

where $\kappa_{||}^0(\bar{Z})$ and $\kappa_\perp^0(\bar{Z})$ are functions that depend on only the ion charge. When the plasma is hot or the initial temperature is discontinuous, the thermal conductivity is the fastest time scale phenomenon. Placing caps on the conductivity may greatly speed up the calculations. This principle applies to viscosity and magnetic resistivity as well.

1.1.2 Magnetic resistivity

The anisotropic magnetic resistivity is given by the tensor

$$\overleftrightarrow{\eta} = \begin{bmatrix} \eta_\perp & -\eta_H & 0 \\ \eta_H & \eta_\perp & 0 \\ 0 & 0 & \eta_{||} \end{bmatrix}$$

where the parallel, perpendicular, and Hall resistivities are

$$\begin{aligned}
\eta_{||} &= \eta_{||}^0(\bar{Z})m_e\nu_e/ne^2 \\
\eta_\perp &= m_e\nu_e/ne^2 \\
\eta_H &= \eta_{||}^0(\bar{Z})m_e\omega_{ce}/ne^2 = \eta_{||}\delta_e.
\end{aligned}$$

The magnetic field changes according to

$$\begin{aligned}
\frac{\partial \mathbf{B}}{\partial t} &= \nabla \times (\overleftrightarrow{\eta}\mathbf{J}) \\
&= \begin{bmatrix} \frac{\partial}{\partial x_\wedge}\left(\eta_{||}J_{||}\right) - \frac{\partial}{\partial x_{||}}\left(\eta_H J_\perp + \eta_\perp J_\wedge\right) \\ \frac{\partial}{\partial x_{||}}\left(\eta_\perp J_\perp - \eta_H J_\wedge\right) - \frac{\partial}{\partial x_\perp}\left(\eta_{||}J_{||}\right) \\ \frac{\partial}{\partial x_\perp}\left(\eta_H J_\perp + \eta_\perp J_\wedge\right) - \frac{\partial}{\partial x_{||}}\left(\eta_\perp J_\perp - \eta_H J_\wedge\right) \end{bmatrix}
\end{aligned}$$

1.1.3 Viscosity

The ion stress tensor is

$$
\begin{aligned}
\Pi_{xx} &= -\frac{\pi_0}{2}(W_{xx}+W_{yy}) - \frac{\pi_1}{2}(W_{xx}-W_{yy}) - \pi_3 W_{xy} \\
\Pi_{yy} &= -\frac{\pi_0}{2}(W_{xx}+W_{yy}) - \frac{\pi_1}{2}(W_{xx}-W_{yy}) - \pi_3 W_{xy} \\
\Pi_{xy} &= \Pi_{yx} = -\pi_1 W_{xy} + \frac{\pi_3}{2}(W_{xx}-W_{yy}) \\
\Pi_{xz} &= \Pi_{zx} = -\pi_2 W_{xz} - \pi_4 W_{yz} \\
\Pi_{yz} &= \Pi_{zy} = -\pi_2 W_{yz} + \pi_4 W_{xz} \\
\Pi_{zz} &= -\eta_0 W_{zz}
\end{aligned}
$$

where the z-axis is defined as parallel to the magnetic field. The values for the ion viscosities are

$$
\begin{aligned}
\pi_0 &= 0.96 nkT_i\tau_i \\
\pi_1 &= (3/10) nkT_i\tau_i/\delta_i^2 \\
\pi_2 &= (6/5) nkT_i\tau_i/\delta_i^2 \\
\pi_3 &= (1/2) nkT_i\tau_i/\delta_i \\
\pi_4 &= nkT_i\tau_i/\delta_i,
\end{aligned}
$$

and the rate of strain tensor is defined as

$$W_{jk} = \frac{\partial v_j}{\partial x_k} + \frac{\partial v_k}{\partial x_j} - \frac{2}{3}\delta_{jk}\nabla\cdot\mathbf{v}.$$

1.1.4 Charge dependence

Many of the Braginskii transport equations contain a numerical factor that depends on $\bar{Z}$, the average charge of each ion. Five values at integer values of $\bar{Z}$ are provided in the Braginskii paper [1].

$\bar{Z}$	1	2	3	4	∞
$\kappa_{\parallel}^0(\bar{Z})$	3.1616	4.890	6.064	6.920	12.471
$\kappa_{\perp}^0(\bar{Z})$	4.664	3.957	3.721	3.604	3.25
$\eta_{\parallel}^0(\bar{Z})$	0.5129	0.4408	0.3965	0.3752	0.2949

Approximate fits can be used to provide an estimate of each factor at non-integer $\bar{Z}$.

$$
\begin{aligned}
\kappa_{\parallel}^0(\bar{Z}) &= 12.47 - 13.92\cdot\exp\left(-0.4023\cdot\bar{Z}^{0.5967}\right) \\
\kappa_{\perp}^0(\bar{Z}) &= 3.250 + 26080\cdot\exp\left(-9.825\cdot\bar{Z}^{0.09648}\right) \\
\eta_{\parallel}^0(\bar{Z}) &= 0.2949 + 0.5285\cdot\exp\left(-0.8845\cdot\bar{Z}^{0.5525}\right)
\end{aligned}
$$

However, it is common to assume that $Z = 1$ because the numerical factors in the Braginskii transport equations vary by only a factor of order unity. This is accomplished by using the default setting for Braginskii_Z_depend.

```
Braginskii_Z_depend = .false.
```

Chapter 2

Magnetohydrodynamics Test Problems

2.1 Anisotropic Conductivity Loop Field Problem

The correct behaviour of the anisotropic conductivity can be checked qualitatively with the 2D loop field problem. The temperature profile is initially specified by the periodic function

$$T(x, y) = T_0 \sin\left(\frac{2\pi x}{L_x}\right) + T_{bkgd}$$

and the magnetic field is everywhere non-zero and pointing in the azimuthal direction $\hat{\phi}$. The magnetic and hydrodynamic fluxes are held constant. Only thermal conduction is performed.

Figure 2.1 shows that when the parallel conductivity is set to $\kappa_{||} = 10^7$ W/(m·K) and the perpendicular conductivity is zero, the thermal gradient quickly becomes zero parallel to the magnetic field. Figure 2.2 likewise shows that when the parallel conductivity is set to $\kappa_{\perp} = 10^7$ and the parallel conductivity is zero, the thermal gradient becomes zero perpendicular to the magnetic field. Since the point $r = 0$ falls on all lines of constant ϕ, the temperature becomes spatially uniform.

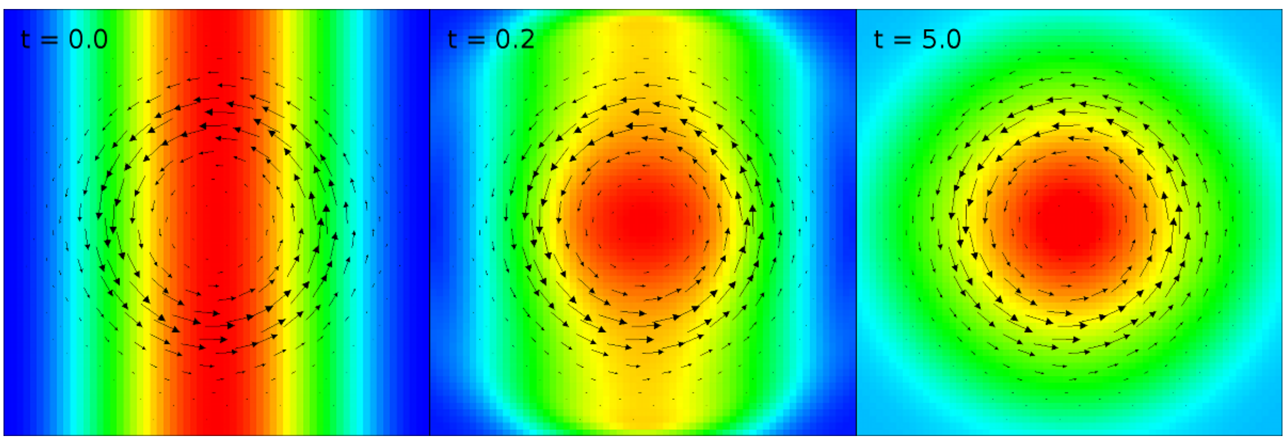

Figure 2.1: Anisotropic thermal conduction in presence of loop field when $\kappa_{||} = 10^7$ and $\kappa_{\perp} = 0$ W/(m·K).

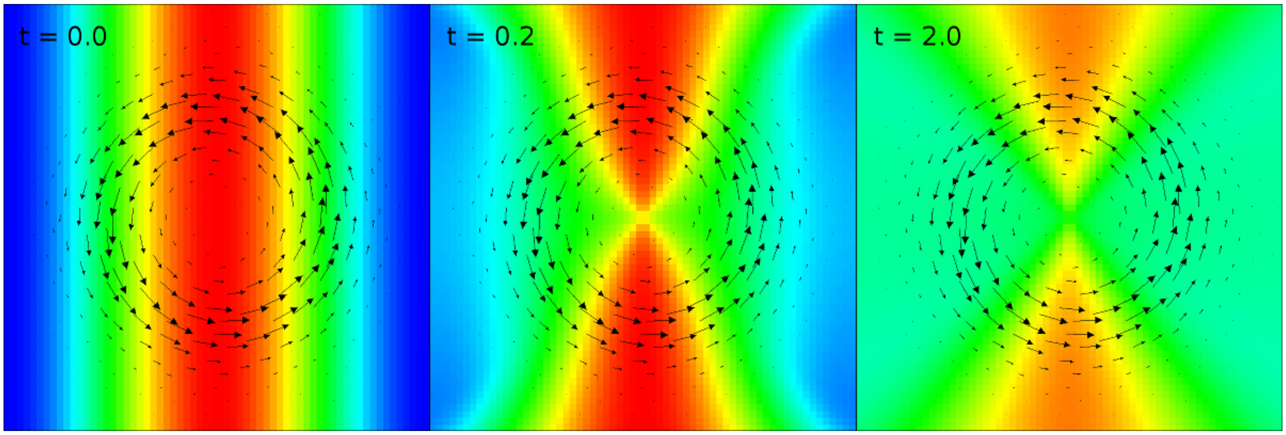

Figure 2.2: Anisotropic thermal conduction in presence of loop field when $\kappa_{||} = 0$ and $\kappa_{\perp} = 10^7$ W/(m·K).

2.2 Anisotropic Conductivity Uniform Field Problem

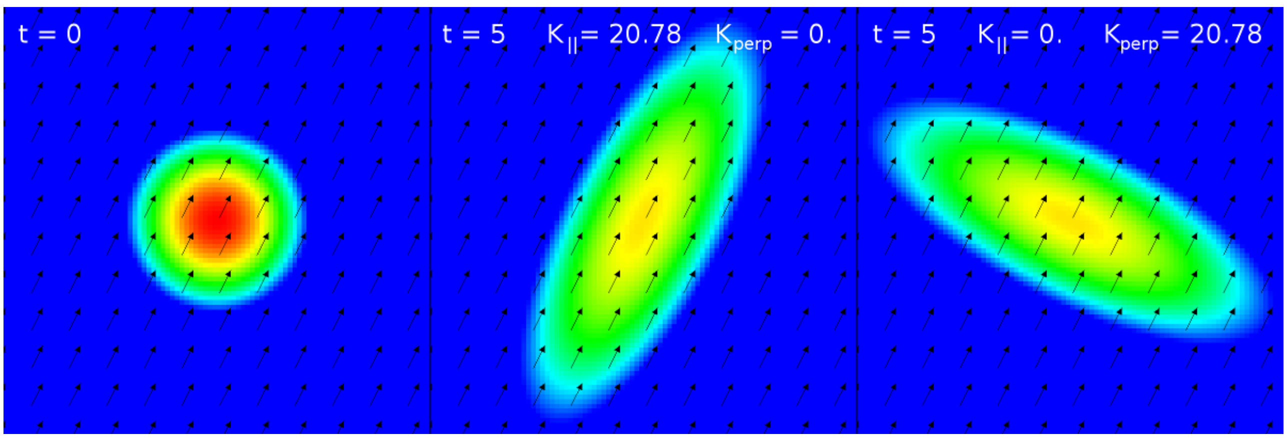

Figure 2.3: Influence of anisotropic thermal conduction on a Gaussian temperature distribution in a uniform field.

2.3 Anisotropic Resistivity Loop Field Problem

2.4 Anisotropic Viscosity Loop Field Problem

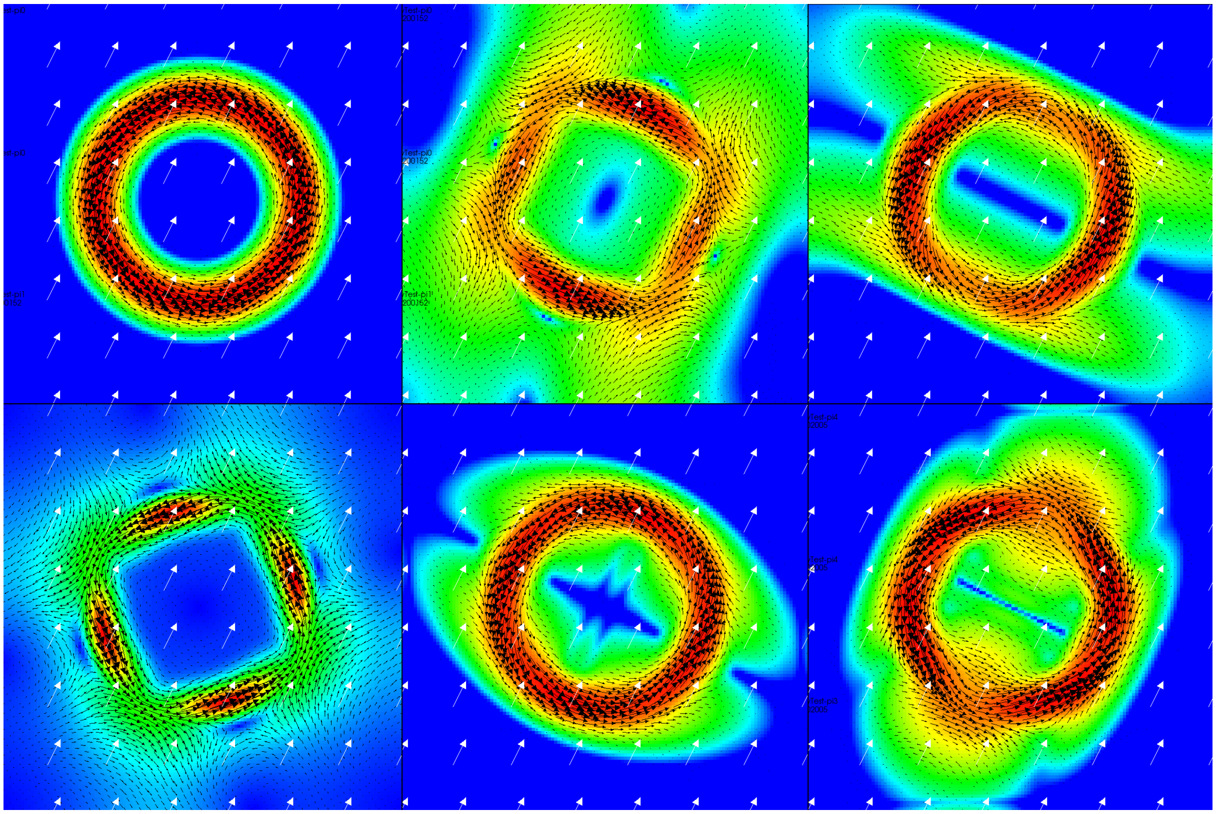

Figure 2.4: Influence of anisotropic viscosity on velocity loop with a Gaussian distribution in a uniform magnetic field. (a) Initial conditions, (b) Case $\pi_0 = 1$ at $t = 0.02$ (c) Case $\pi_1 = 1$ at $t = 0.02$ (d) Case $\pi_2 = 1$ at $t = 0.02$ (e) Case $\pi_3 = 1$ at $t = 0.02$ (f) Case $\pi_4 = 1$ at $t = 0.02$.

2.5 Sources Cited

[1] S.I. Braginskii, Transport Processes in a Plasma, in Reviews of Plasma Physics (Consultants Bureau, New York NY, 1965), Vol. 1, p. 205.