I. hy_uhd_prim2flx.F90

Consider the momentum and induction equations:

$$\frac{\partial \rho u}{\partial t} + \nabla \bullet (\rho u u - BB) + \nabla \bullet ptot = 0 \quad (1)$$

$$\frac{\partial B}{\partial t} + \nabla \bullet (uB - Bu) = 0 \qquad (2)$$

The x-components of the above two equations are:

$$\frac{\partial \rho u_x}{\partial t} + \frac{\partial \rho u_x u_x}{\partial x} + \frac{\partial \rho u_y u_x}{\partial y} + \frac{\partial \rho u_z u_x}{\partial z} - \frac{\partial B_x B_x}{\partial x} - \frac{\partial B_y B_x}{\partial y} - \frac{\partial B_z B_x}{\partial z} + \frac{\partial ptot}{\partial x} = 0 \quad (3)$$

$$\frac{\partial B_x}{\partial t} + \frac{\partial (u_y B_x - u_x B_y)}{\partial y} + \frac{\partial (u_z B_x - u_x B_z)}{\partial z} = 0 \qquad (4)$$

In the subroutine hy_uhd_prim2flx, for dir==DIR_X the fluxes are calculated as followings:

F(F02XMOM_FLUX) = $\rho u_x u_x - B_x B_x + ptot$

F(F03YMOM_FLUX) = $\rho u_y u_x - B_y B_x$

F(F04ZMOM_FLUX) = $\rho u_z u_x - B_z B_x$

#ifdef FLASH_USM_MHD

F(F06MAGX_FLUX) = 0.0

#endif

F(F07MAGY_FLUX) = $u_x B_y - u_y B_x$

F(F08MAGZ_FLUX) = $u_x B_z - u_z B_x$

The fluxes F(F02XMOM_FLUX), F(F03YMOM_FLUX) and F(F04ZMOM_FLUX) are coded based on equation (3). However, the F(F07MAGY_FLUX) and F(F08MAGZ_FLUX) are written in an opposite way with equation (4). I think if (3) is coded properly, then (4) has to be coded as followings:

F(F07MAGY_FLUX) = $u_y B_x - u_x B_y$

F(F08MAGZ_FLUX) = $u_z B_x - u_x B_z$

I don't know what I thought is right or wrong? If it is wrong then can you explain why the magnetic fields are oppositely coded with equation (4).

II. hy_uhd_addViscousFluxes.F90

The viscosity term is added into the momentum equation is as following:

$$\frac{\partial \rho u}{\partial t} + \nabla \cdot (\rho u u - BB) + \nabla \cdot ptot - \nabla \cdot \tau = 0 \quad (5)$$

where

$$\tau = \mu\left[(\nabla u) + (\nabla u)^T - \frac{2}{3}(\nabla \cdot u)I\right].$$

The x-component of equation (5) can be written as

$$\frac{\partial \rho u_x}{\partial t} + [\dots] + \frac{\partial}{\partial x}\mu\left(-\frac{4}{3}\frac{\partial u_x}{\partial x} + \frac{2}{3}\frac{\partial u_y}{\partial y} + \frac{2}{3}\frac{\partial u_z}{\partial z}\right) + \frac{\partial}{\partial y}\mu\left(-\frac{\partial u_x}{\partial y} - \frac{\partial u_y}{\partial x}\right) + \frac{\partial}{\partial z}\mu\left(-\frac{\partial u_x}{\partial z} - \frac{\partial u_z}{\partial x}\right) = 0 \quad (6)$$

Where $[\dots] \equiv \frac{\partial \rho u_x u_x}{\partial x} + \frac{\partial \rho u_y u_x}{\partial y} + \frac{\partial \rho u_z u_x}{\partial z} - \frac{\partial B_x B_x}{\partial x} - \frac{\partial B_y B_x}{\partial y} - \frac{\partial B_z B_x}{\partial z} + \frac{\partial ptot}{\partial x}.$

The viscosity term is controlled by subroutine hy_uhd_addViscousFluxes. Consider the case dir==DIR_X, the viscous flux is coded as following:

```
Flux(F02XMOM_FLUX:F04ZMOM_FLUX) = Flux(F02XMOM_FLUX:F04ZMOM_FLUX)&
        -idx*mu_loc*(U(VELX_VAR:VELZ_VAR,ix,iy,iz)-U(VELX_VAR:VELZ_VAR,ix-1,iy,iz))

Flux(F02XMOM_FLUX) = Flux(F02XMOM_FLUX)-idx*mu_loc*(U(VELX_VAR,ix,iy,iz)-
U(VELX_VAR,ix-1,iy,iz))/3.
```

These two command lines can be rewritten as

Flux(F02XMOM_FLUX) = Flux(F02XMOM_FLUX) - $\mu\frac{4}{3}\frac{\partial u_x}{\partial x}$

Flux(F03YMOM_FLUX) = Flux(F03YMOM_FLUX) - $\mu\frac{\partial u_y}{\partial x}$

Flux(F04ZMOM_FLUX) = Flux(F04ZMOM_FLUX) - $\mu\frac{\partial u_z}{\partial x}$

```
#if NDIM >= 2
    !! d/dz=0
    Flux(F02XMOM_FLUX) = Flux(F02XMOM_FLUX)+idy*mu_loc*&
              (U(VELY_VAR, ix ,iy+1,iz)-U(VELY_VAR, ix ,iy-1,iz)+ &
               U(VELY_VAR,ix-1,iy+1,iz)-U(VELY_VAR,ix-1,iy-1,iz))/3.
```

This line can be rewritten as

Flux(F02XMOM_FLUX) = Flux(F02XMOM_FLUX) + $\mu\frac{4}{3}\frac{\partial u_y}{\partial y}$????

I think that we should multiply 0.5 to get the exact form of Flux(F02XMOM_FLUX)

Flux(F02XMOM_FLUX) = Flux(F02XMOM_FLUX) + $\mu \frac{2}{3}\frac{\partial u_y}{\partial y}$

```
#if NDIM == 3
        Flux(F02XMOM_FLUX) = Flux(F02XMOM_FLUX)+0.5*idz*mu_loc*&
                (2./3.)*(U(VELZ_VAR, ix ,iy,iz+1)-U(VELY_VAR, ix ,iy,iz-1)+ &
                         U(VELZ_VAR,ix-1,iy,iz+1)-U(VELY_VAR,ix-1,iy,iz-1))/2.
```

This line can be rewritten as

Flux(F02XMOM_FLUX) = Flux(F02XMOM_FLUX) +

$$\mu \frac{2}{3}\left(\frac{u_z(ix,iyiz+1)-u_y(ix,iy,iz-1)}{\Delta z}+\frac{u_z(ix-1,iy,iz+1)-u_y(ix-1,iy,iz-1)}{\Delta z}\right)$$

I think that this line in not coded correctly. It should be like this

Flux(F02XMOM_FLUX) = Flux(F02XMOM_FLUX) +

$$\mu \frac{1}{3}\left(\frac{u_z(ix,iy,iz+1)-u_z(ix,iy,iz-1)}{2\Delta z}+\frac{u_z(ix-1,iy,iz+1)-u_z(ix-1,iy,iz-1)}{2\Delta z}\right)$$

= Flux(F02XMOM_FLUX) + $\mu \frac{2}{3}\frac{\partial u_z}{\partial z}$

If what I thought is wrong, please give explanations. Thank you very much.