The Center for Astrophysical Thermonuclear Flashes

Various Hydro Solvers in FLASH3

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FLASH3 Tutorial
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Academic Strategic Alliances Program (ASAP)
Center at The University of Chicago

Hydro Solvers in FLASH3

- In FLASH3, "Hydro" unit houses more than one usual gas dynamics solver:
  - Pure hydrodynamics (i.e., gas dynamics) solvers (PPM & MUSCL-Hancock)
  - Magnetohydrodynamics(MHD) solvers (Unsplit Staggered Mesh & 8-wave)
  - Relativistic hydrodynamics (RHD) solver

- The Hydro unit is organized into two different subunits depending on how you treat multidimensional flux updates:
  - Operator (dimensional) Splitting (Strang, 1968) vs. Unsplit (Colella, 1990; Lee & Deane, 2009)
    - source/Hydro/HydroMain/split (PPM, 8-wave MHD, RHD)
    - source/Hydro/HydroMain/unsplit (Staggered Mesh MHD, MUSCL-Hancock pure-Hydro)

- All these five major different solvers are based on high-order Godunov (1959) method which involves:
  - Finite volume method
  - Predictor-corrector
  - Riemann problem
  - Explicit time advancement

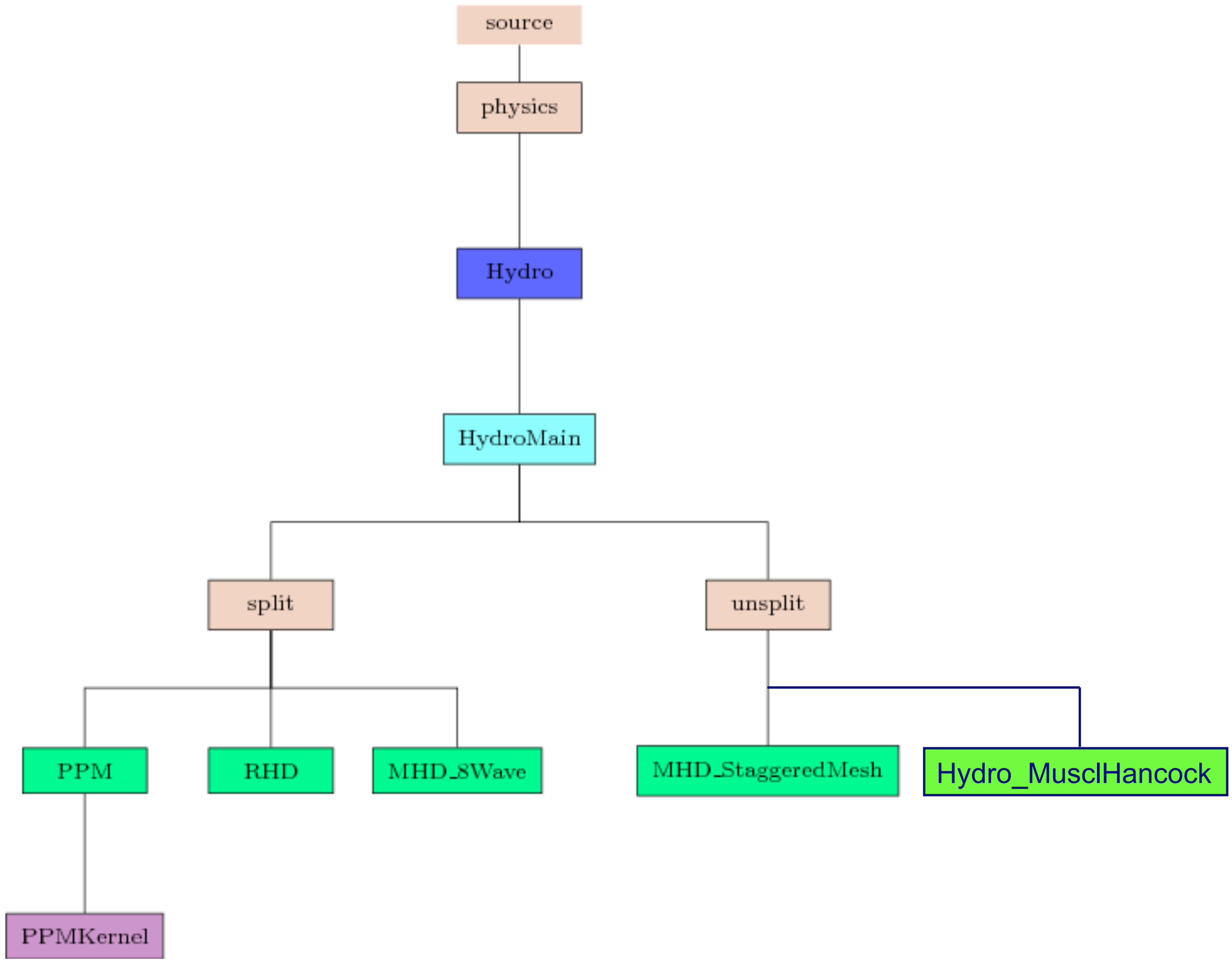
Physics of Hydro Solvers

- Pure hydrodynamics solvers (PPM & MUSCL-Hancock)
  - Compressible reactive gas dynamics
  - Can solve a broad range of (astro)physical problems

- MHD solvers (Unsplit Staggered Mesh & 8-wave)
  - flows of conducting fluids (ionized gases, liquid metals) in presence of magnetic fields
  - Plasma is a completely ionized gas, consisting of freely moving positively charged ions (or nuclei) and negatively charged electrons
  - Lorentz forces act on charged particles and change their momentum and energy. In return, particles alter strength and topology of magnetic fields.
  - A valid macroscopic model of magnetized plasma → MHD

- Relativistic hydrodynamics solver (RHD)
  - A wide variety of astrophysical flows exhibit relativistic behavior
  - accretion around compact objects, jets in extragalactic radio sources, pulsar winds, gamma ray bursts

Hydro Unit in FLASH3



Operator Splitting vs. Unsplit Formulations

$$\frac{\partial U}{\partial t} + \nabla \cdot \mathrm{Flux} = 0$$

$$\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} = 0$$

Operator Splitting vs. Unsplit Formulations

$$\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} = 0$$

Splitting

$$X^{\Delta t}: \left.\begin{array}{l} \mathrm{PDE}: U_t + F(U)_x = 0 \\ \mathrm{IC}: \quad U^n \end{array}\right\} \xrightarrow{\Delta t} U^{n+1/2}$$

$$Y^{\Delta t}: \left.\begin{array}{l} \mathrm{PDE}: U_t + G(U)_y = 0 \\ \mathrm{IC}: \quad U^{n+1/2} \end{array}\right\} \xrightarrow{\Delta t} U^{n+1}$$

1st order Strang Splitting

$$U^{n+1} = X^{\Delta t} Y^{\Delta t} U^n$$

2nd order Strang Splitting

$$U^{n+1} = \left(X^{\Delta t/2} Y^{\Delta t/2}\right)\left(Y^{\Delta t/2} X^{\Delta t/2}\right) U^n$$

Unsplit

$$\mathrm{PDE}: U_t + F(U)_x + G(U)_y = 0$$

$$\mathrm{IC}: \quad U(x, y, t^n) = U^n$$

Splitting vs. Unsplit

source/Hydro/HydroMain

/split

- PPM
- MHD_8Wave
- RHD

- Easy to implement
- Robust, stable, efficient
- Less memory and computation
- Good enough for gas dynamics
- Bad for MHD

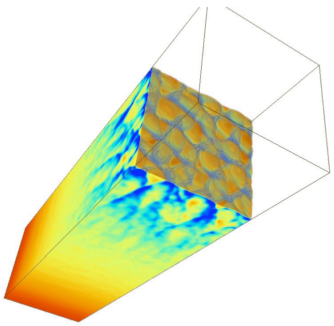
/unsplit

- Hydro_MusclHancock
- MHD_StaggeredMesh

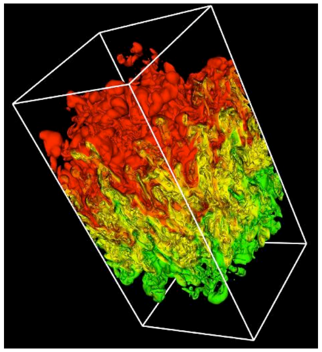
- Not easy to implement a robust & stable solver
- More memory and computation
- Good for preserving symmetries
- Very important for MHD!

Hydro Solvers Primer – Split solvers

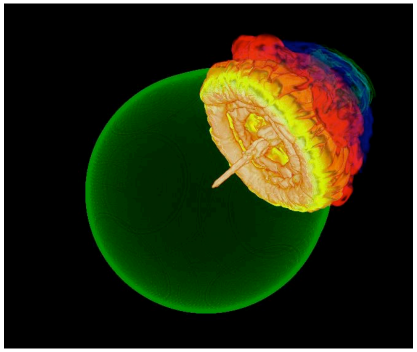
- **Piecewise-parabolic method solver (PPM) (Fryxell et al., 2000)**
  - Parabolic interpolation of data over each cell (Colella and Woodward, 1984)
    - (Ideally) 3rd order, (formally) 2nd order, (practically) 1st order in shocks and discontinuities in spatial discretization
    - High resolution with accuracy (smooth flows)
    - Monotonocity enforcement, interpolant flattening, steepening of contact discontinuities
  - 2nd order in explicit time evolution using operator splitting formulation



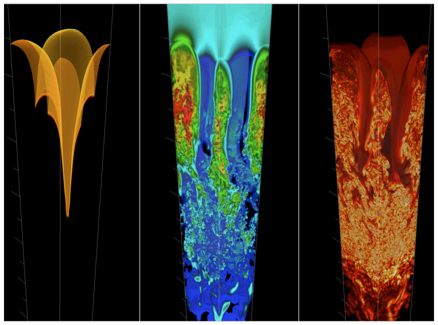
Cellular detonation



Rayleigh-Taylor instability



Gravitationally confined detonation

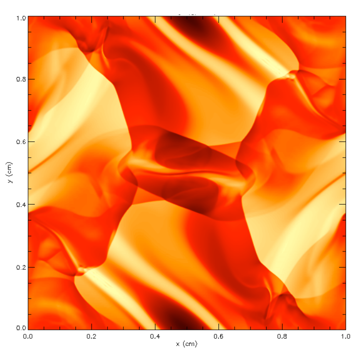


Turbulent Nuclear Burning

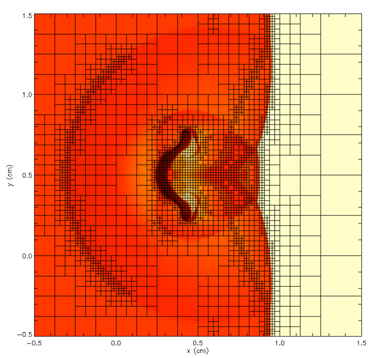
Hydro Solvers Primer – Split solvers

- **MHD 8-wave solver (Timur Linde, 1999)**
  - Monotone Upstream-centered Scheme for Conservation Laws (MUSCL) approach (Van Leer, 1977)
    - 2nd order in space, 2nd order in time
    - Magnetic monopoles (8th wave) are convected away, rather than accumulated (Powell et al., 1998) (i.e., $\nabla \cdot B \neq 0$)
    - Non-conservative formulation of the MHD governing equations
    - Incorrect jump conditions and incorrect propagation speeds across discontinuites
    - Robust and accurate (as compared to the basic conservative scheme)

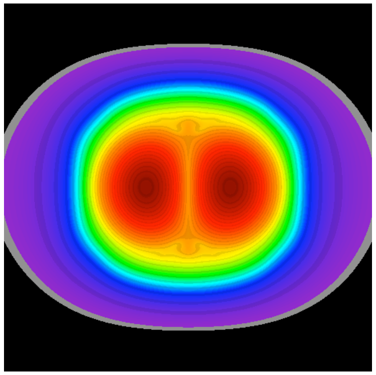
Orszag-Tang



Shock-Cloud Interaction



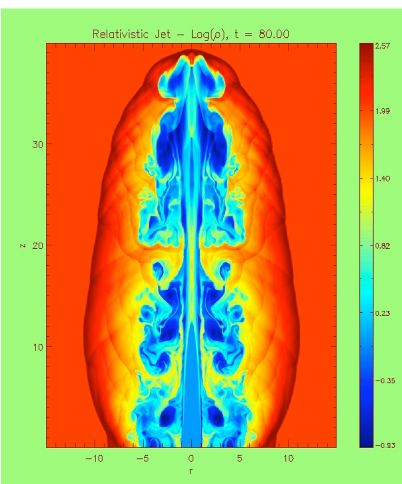
Magnetic reconnection



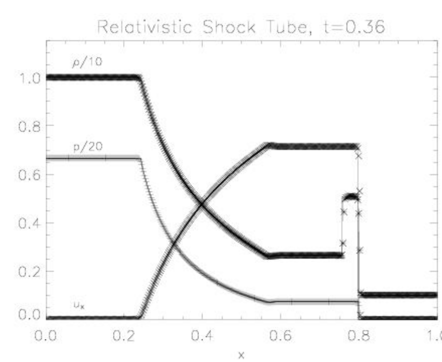
Magnetic RT

Hydro Solvers Primer – Split solvers

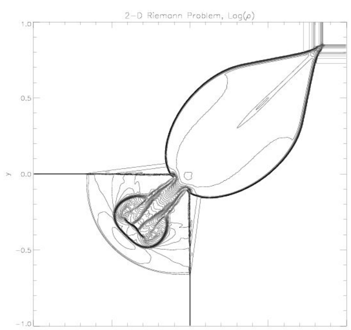
- **Special Relativistic solver (RHD) (A. Mignone, 2004)**
  - PPM (3^{rd} order in space) and PLM (2^{nd} order) interpolations
  - 2^{nd} order in explicit time evolution using operator splitting formulation
  - (special) relativistic effects are twofold:
    - kinematical, $v \sim c$ ($\gamma = 1/(1 - v^2)^{1/2} >> 1$)
    - thermodynamical, $c_s \sim c$
  - Relativistic flows with $\gamma > (3/2)^{1/2}$ are always <u>supersonic</u>, and therefore shock-capturing methods are essential (Martí and Müller, 2003)



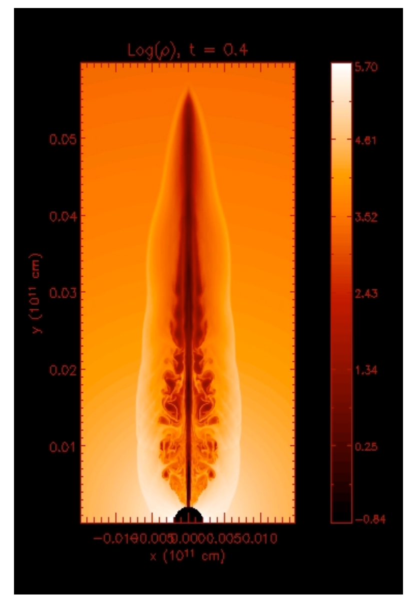
γ =10 Jet



Relativistic Shock tube



2-D Riemann Problem

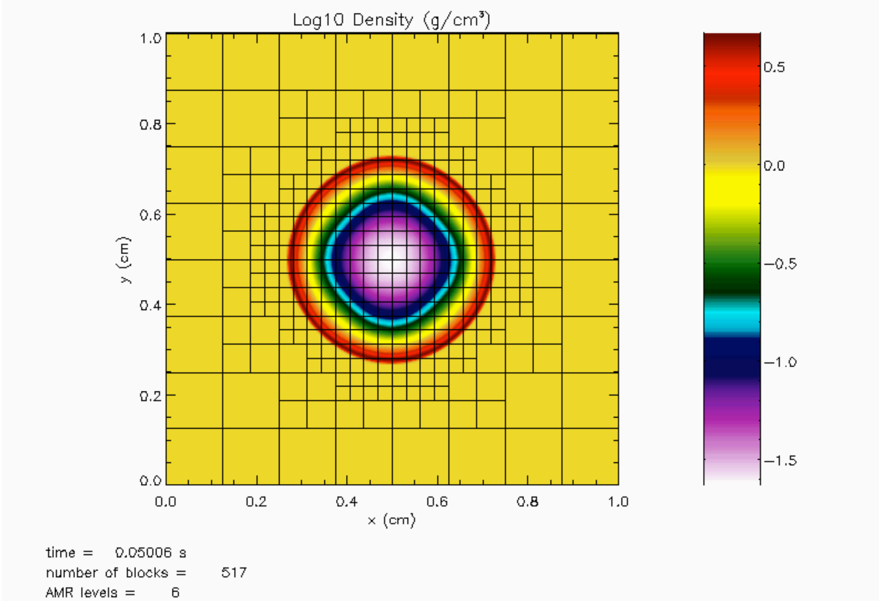


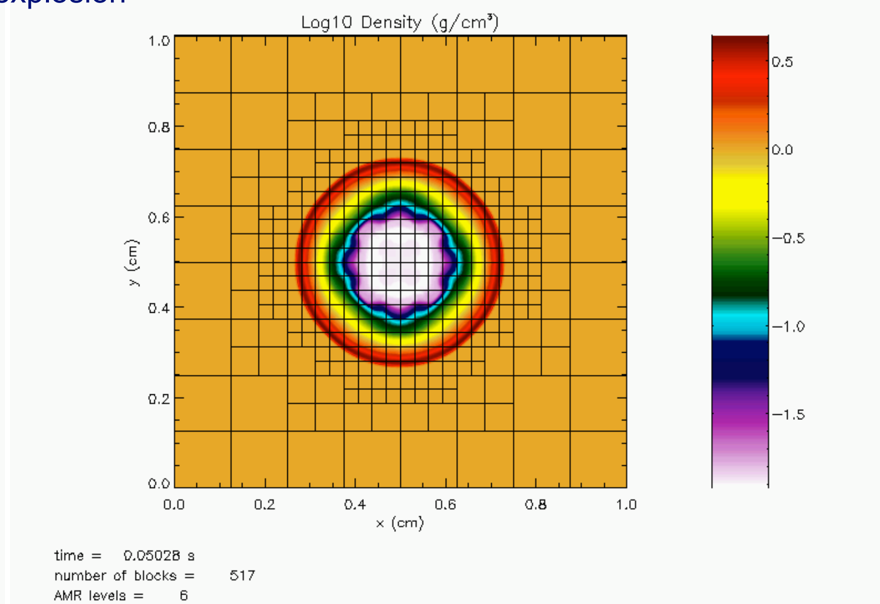
Jet through collapsars (GRB), $\gamma \sim 50$

Hydro Solvers Primer – Unsplit solvers

- **Unsplit pure-Hydro solver (Lee, 2009)**
  - Reduced version of USM-MHD solver without magnetic and electric fields
  - 2^{nd} order MUSCL-Hancock in space and time
  - Preserves better flow symmetries (Roe solver + Carbuncle instability fix)

Sedov explosion



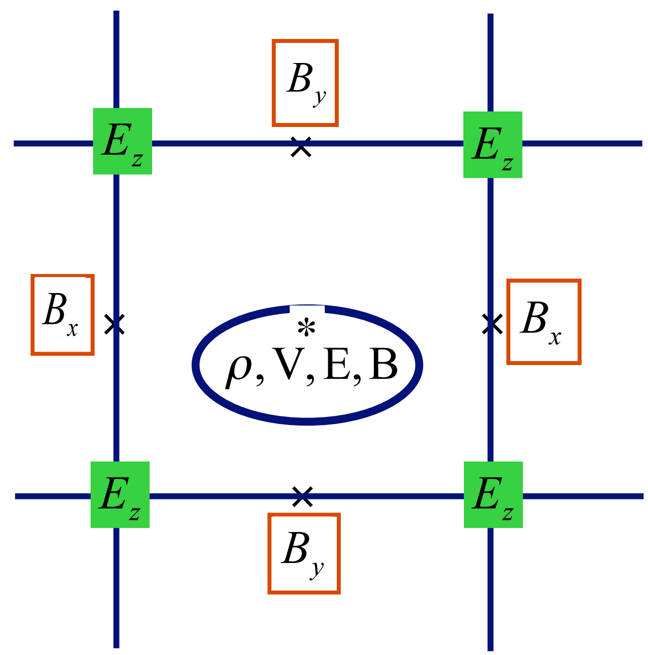


Unsplit MUSCL-Hancock

Split PPM

Hydro Solvers Primer – Unsplit solvers

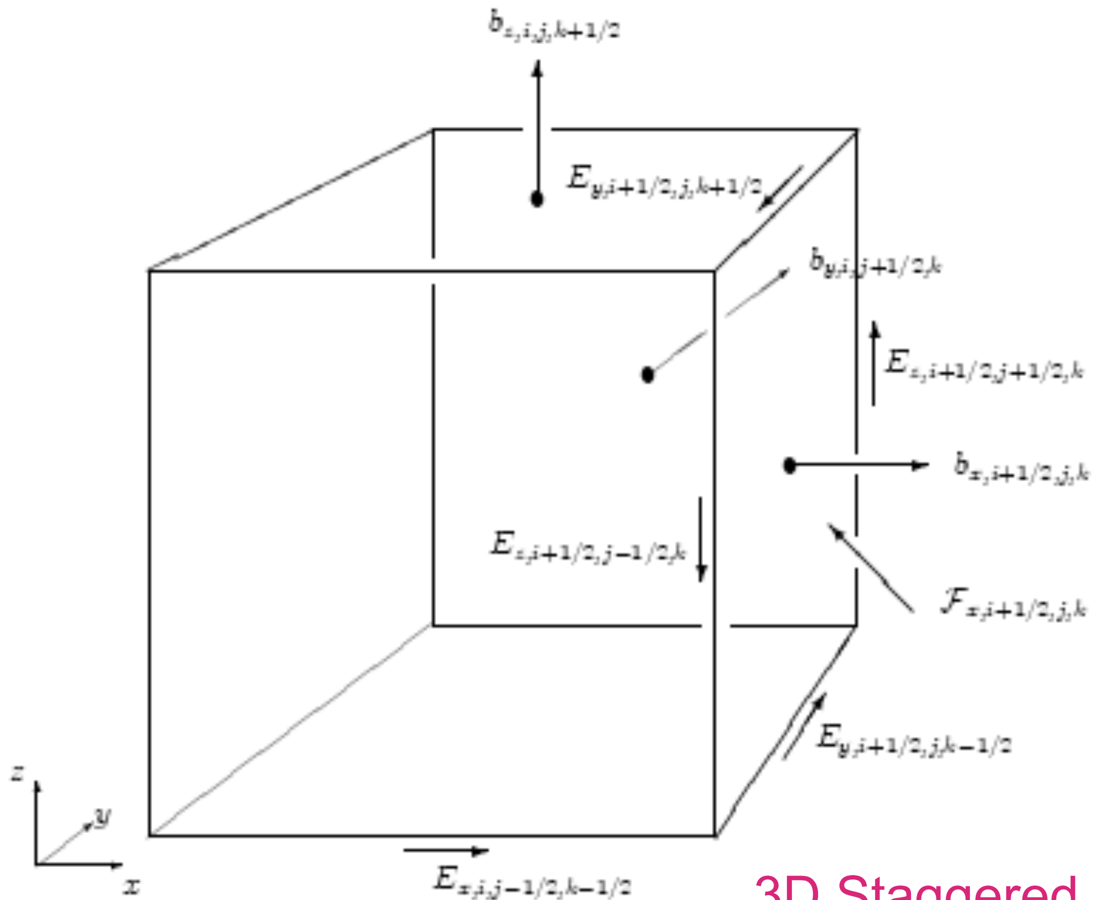
- **Unsplit Staggered Mesh (USM) MHD solver (Lee, 2006; Lee and Deane 2009)**
  - A very efficient new data reconstruction algorithm
    - MUSCL-Hancock (2nd order in space) type characteristic tracing method
    - No extra Riemann problems for transverse fluxes!
  - A new way of treating multidimensional MHD source terms in unsplit formulation
  - Constrained Transport (CT) algorithm (Evans and Hawley, 1988; Balsara and Spicer, 1999) for induction equations to maintain $\nabla \cdot \mathrm{B} = 0$ on a staggered grid
    - cell-centered, face-centered, corner (edge-centered) variables
  - Enhanced solution accuracy in calculating electric fields for the induction equations (modified electic field construction)
    - Added proper amount of numerical dissipation – important!



2D staggered mesh

Hydro Solvers Primer – Unsplit solvers

- Unsplit Staggered Mesh (USM) MHD solver (Lee, 2006; Lee and Deane 2009)



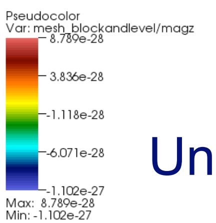
3D Staggered Mesh

Why unsplit formulation is necessary for MHD?

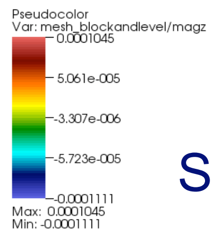
- Operator splitting MHD schemes cannot avoid erroneous growth in B_z :

$$\frac{\partial B_z}{\partial t} + B_z\frac{\partial u}{\partial x} - B_x\frac{\partial w}{\partial x} - \boxed{w\frac{\partial B_x}{\partial x}} + u\frac{\partial B_z}{\partial x} + B_z\frac{\partial v}{\partial y} - B_y\frac{\partial w}{\partial y} - \boxed{w\frac{\partial B_y}{\partial y}} + v\frac{\partial B_z}{\partial y} = 0$$

$$\boxed{w\nabla\cdot\mathbf{B} = w(\Delta B_{x,i}/\Delta x + \Delta B_{y,j}/\Delta y).}$$

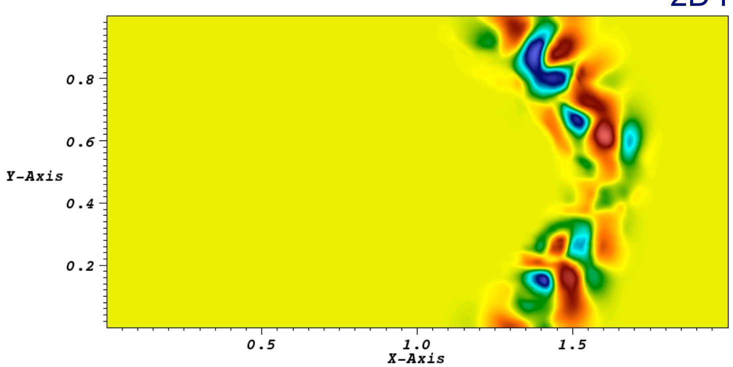


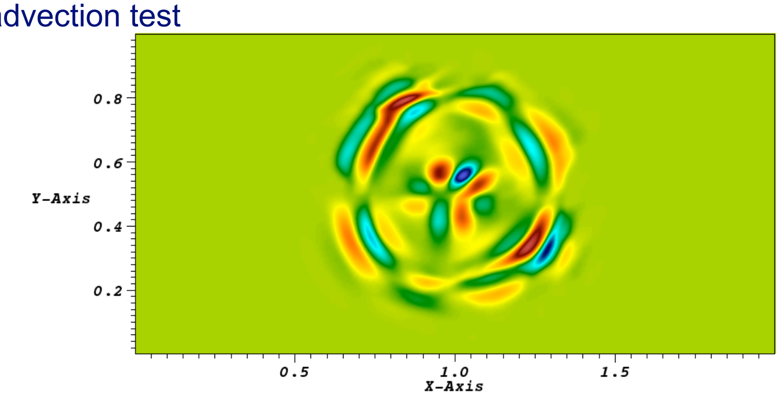
Unsplit Staggered Mesh solver



Split 8-wave solver

2D Field Loop advection test





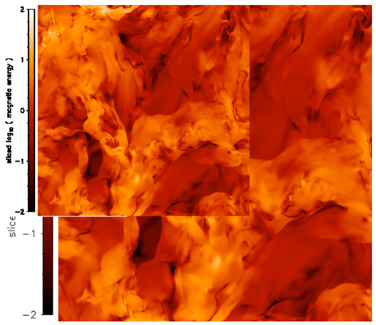
More on the USM-MHD solver

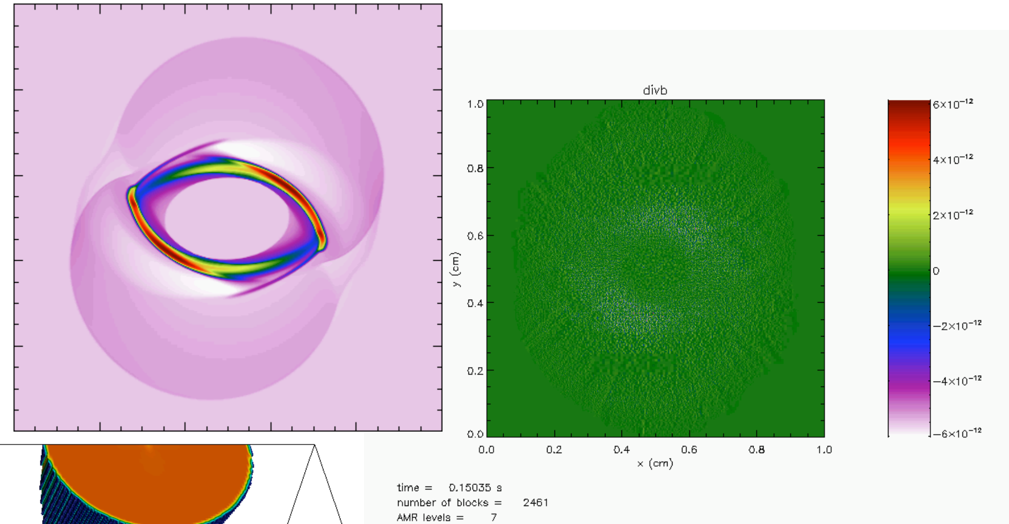
- Physics
  - Ideal and non-ideal flows
    - Magnetic resistivity, themal conductivity, and viscosity
  - EOS
    - Ideal gamma, multiple gamma, Helmholtz (degenerate EOS)
  - Gravity
  - Multiple species, particles
  - Well tested for wide ranges of plasma flows: $10^{-6} < \beta\left(\equiv p/B_p\right) < 10^{6}$

- Implementations and algorithms
  - Riemann solvers
    - Roe (default), HLLE, HLLC, HLLD (robust and accurate, suggested for most plasma flows)
    - Carbuncle, even-odd instability fix for Roe solver
  - Strong shock-rarefaction detect algorithm (Balsara)
  - Various slope limiters (Minmod, MC, Van Leer, hybrid)
  - Two prolongation methods of divergence-free B fields on AMR
  - Use of face-centered variables, and edge-centered variable
  - Wide ranges of CFL limit: CFL < 1 for 1D, 2D and 3D

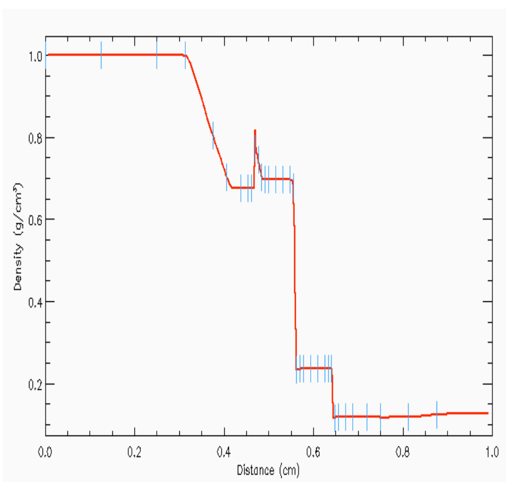
Applications

High Mach Number MHD Turbulence

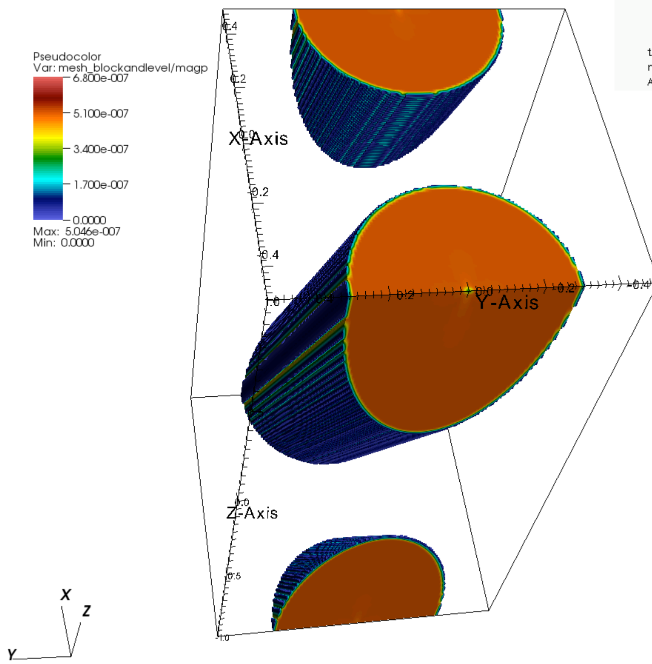




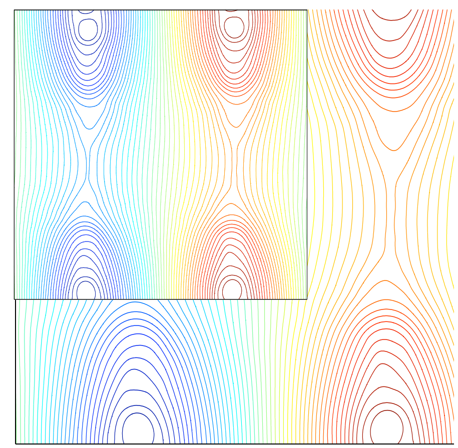
Rotor



Brio-Wu MHD shock-tube



3D Field Loop advection



Magnetic Reconnection

How to Setup a New Problem

- MHD Simulation (especially with the USM solver) should be located in
  - source/Simulation/SimulationMain/magnetoHD/
  - Special prolongation for face-centered variables

- Create Simulation_initBlock.F90 exactly as you would do for hydro. Just do not forget to set magnetic field variables (both cell-centered and cell face -centered) in the initialization routine.
  - Magnetic fields need to satisfy $\nabla \cdot \mathrm{B} = 0$

- Do not add magnetic pressure to total specific energy, because FLASH EOS routines assume a specific expression for it.

- Create Config and flash.par files for your own Simulation directory.

- Special care in writing custom boundary conditions in Grid_bcApplyToRegionSpecialized.F90.

- Write custom functions and do not forget to add them to Makefile. Such custom functions in your Simulation directory will override other standard implementations.

Example – Config

```
#       Configuration file for Orszag Tang MHD vortex problem
#       (Orszag and Tang, J. Fluid Mech., 90:129--143, 1979)

REQUIRES physics/Hydro/HydroMain
REQUIRES physics/Eos/EosMain/Gamma

USESETUPVARS withParticles

IF withParticles
   PARTICLETYPE passive INITMETHOD lattice MAPMETHOD quadratic

   REQUIRES Particles/ParticlesMain
   REQUESTS IO/IOMain
   REQUESTS IO/IOParticles
   REQUESTS Particles/ParticlesMapping/Quadratic
   REQUESTS Particles/ParticlesInitialization/Lattice
ENDIF

D   tiny            Threshold value used for numerical zero
PARAMETER tiny         REAL     1.e-16

# -----------------  For Resistive MHD setup -----------------#
#REQUIRES physics/materialProperties/Conductivity/ConductivityMain/Constant-diff
#REQUIRES physics/materialProperties/Viscosity/ViscosityMain
#REQUIRES physics/materialProperties/MagneticResistivity/MagneticResistivityMain
#REQUIRES physics/sourceTerms/Diffuse/DiffuseMain

#VARIABLE vecz                  # vector potential Az
#------------------ End of Resistive MHD setup ----------------#
```

Example – flash.par

```
#       DivB control switch
killdivb          = .true.

#       Flux Conservation for AMR
flux_correct      = .true.


## -------------------------------------------------------------##
##  SWITCHES SPECIFIC TO THE UNSPLIT STAGGERED MESH MHD SOLVER  ##
#       I. INTERPOLATION SCHEME:
order             = 2      # Interpolation order (First/Second order)
slopeLimiter      = "mc"   # Slope limiters (minmod, mc, vanLeer, hybrid, limited)
LimitedSlopeBeta= 1.       # Slope parameter for the "limited" slope by Toro
charLimiting      = .true. # Characteristic limiting vs. Primitive limiting


#       II. MAGNETIC(B) AND ELECTRIC(E) FIELDS:
E_modification    = .true.          # High order algorithm for E-field construction
energyFix         = .true.          # Update magnetic energy using staggered B-fields
ForceHydroLimit = .false.           # Pure Hydro Limit (B=0)
prolMethod        = "injection_prol" # Prolongation method (injecton_prol, balsara_prol)


#       III. RIEMANN SOLVERS:
RiemannSolver     = "Roe"       # Roe, HLL, HLLC, HLLD, LF
shockInstabilityFix = .false.   # Carbuncle instability fix for the Roe solver
entropy           = .false.     # Entropy fix for the Roe solver


#       IV. STRONG SHOCK HANDELING SCHEME:
shockDetect       = .false.     # Shock Detect for numerical stability
## -------------------------------------------------------------##
```

Example – setup lines

- ./setup magnetoHD/OrszagTang -auto -2d +usm (+8wave) -opt (-debug) –objdir=OT2D –with-unit=Particles +pm3 (+pm4dev) -nxb=8 -nyb=8
  - lrefine_min = 1, lrefine_max = 6
- ./setup magnetoHD/Rotor -auto -2d +usm -opt +ug –nxb=200 –nyb=300
  - iProcs = 2, jProcs = 2
- ./setup magnetoHD/Rotor -auto -2d +usm -opt +nofbs
  - iGridSize = 400, jGridSize = 600, iProcs = 2, jProcs = 2
- More shortcuts can be found in
  - /bin/setup_shortcuts.txt
  - Users can add their own customized shortcut(s) by editing the file

Example – setup lines

/bin/setup_shortcuts.txt

```
# Choice of Grid
grid:-unit=Grid:
ug:+grid:Grid=UG:
pm2:+grid:Grid=PM2:
pm40:+grid:Grid=PM40:
pm3:+pm40
pm4dev:+grid:Grid=PM4DEV:ParameshLibraryMode=True

# Choice of MHD solver
# NOTE: The 8wave mhd solver only works with the native interpolation.
8wave:--with-unit=physics/Hydro/HydroMain/split/MHD_8Wave:+grid:-gridinterpolation=native

# NOTE: If pure hydro mode used with the USM solver, add +pureHydro in setup
usm:--with-unit=physics/Hydro/HydroMain/unsplit/MHD_StaggeredMesh:--without-unit=physics/Hydro/HydroMain/split/MHD_8Wave
pureHydro:physicsMode=hydro
unsplitHydro:--with-unit=physics/Hydro/HydroMain/unsplit/Hydro_MusclHancock
```

Tips on setting up your problems

- Simulation's Config file contains all default runtime parameter values specific to your problem
- default.par is also generated in your object directory
- Flash.h, setup_units, setup_vars, setup_params, etc
- Always helpful to read FLASH user's guide
- Further read/refer references to understand the various roles/effects on using different choices of runtime parameters
    - e.g., Roe vs. HLL-type Riemann solvers
- Use "-debug" for testing
- Many (simple) issues come from wrong initializations and wrong boundary conditions
    - Simplify your issues as much as possible
    - Detailed description on your issues/bugs is always welcome
        - Your simulation files, flash.par, log files, boundary conditions, compilers, setup lines, etc.

Tips in general

- Make sure you know your problem
  - Literature research
  - IC, BC, units
  - Any working example?
  - Working on FLASH3

- Comments, comments, and comments – keep your journal

- Useful books and references:
  - LeVeque, Finite volume methods for hyperbolic problems
  - Toro, Riemann solvers and numerical methods for fluid dynamics
  - Laney, Computational gas dynamics
  - Goedbloed and Poedts, Principles of Magnetohydrodynamics
  - NRL plasma formulary
  - FLASH user's guide

- Ask around good people!

Supplementary slides

Ideal MHD Equations

$$\frac{\partial}{\partial t}\begin{pmatrix} \rho \\ \mathbf{V} \\ \rho E \\ \mathbf{B} \end{pmatrix} + \nabla \cdot \begin{pmatrix} \rho \mathbf{V} \\ \rho \mathbf{V}\mathbf{V} + (p + \frac{B^2}{2})\bar{\bar{\mathbf{I}}} - \mathbf{B}\mathbf{B} \\ \mathbf{V}(\rho E + p + \frac{B^2}{2}) - \mathbf{B}(\mathbf{V}\cdot\mathbf{B}) \\ \mathbf{V}\mathbf{B} - \mathbf{B}\mathbf{V} \end{pmatrix} = 0$$

Major Properties:

- MHD equations form a hyperbolic system ➔ Seven families of waves (entropy, Alfvén and fast and slow magnetoacoustic waves).
- Convex space of physically admissible variables if convex EOS.
- Non-convex flux function ➔ Multiple degeneracies in the eigensystem, possibility of compound waves, shock evolutionarity concerns.

♪ **Important to remember**: Fluid (Euler) equations <u>are not</u> the limiting case of MHD equations in the B $\rightarrow$ 0 case in strict mathematical sense.

♪

MHD Equations in FLASH3

$$
\begin{aligned}
\frac{\partial \rho}{\partial t} &+ \nabla \cdot (\rho \mathbf{v}) = 0 \\
\frac{\partial \rho \mathbf{v}}{\partial t} &+ \nabla \cdot (\rho \mathbf{v}\mathbf{v} - \mathbf{B}\mathbf{B}) + \nabla p_* = \rho \mathbf{g} + \nabla \cdot \tau \\
\frac{\partial \rho E}{\partial t} &+ \nabla \cdot (\mathbf{v}(\rho E + p_*) - \mathbf{B}(\mathbf{v} \cdot \mathbf{B})) = \rho \mathbf{g} \cdot \mathbf{v} + \nabla \cdot (\mathbf{v} \cdot \tau + \sigma \nabla T) + \nabla \cdot (\mathbf{B} \times (\eta \nabla \times \mathbf{B})) \\
\frac{\partial \mathbf{B}}{\partial t} &+ \nabla \cdot (\mathbf{v}\mathbf{B} - \mathbf{B}\mathbf{v}) = -\nabla \times (\eta \nabla \times \mathbf{B})
\end{aligned}
$$

where

$$
\begin{aligned}
p_* &= p + \frac{B^2}{2}, \\
E &= \frac{1}{2} v^2 + \epsilon + \frac{1}{2} \frac{B^2}{\rho}, \\
\tau &= \mu \left((\nabla \mathbf{v}) + (\nabla \mathbf{v})^T - \frac{2}{3} (\nabla \cdot \mathbf{v}) \right)
\end{aligned}
$$

- Advective terms are discretized using slope-limited TVD scheme.
- Diffusive terms are discretized using central finite differences.
- Time integration is done using one-stage Hancock scheme.
- Directions are either split or unsplit.

Overview on Numerical MHD codes

- Numerical MHD
  - Finite difference method (e.g., ZEUS code by Stone & Norman, 1992)
  - High order Godunov method – high resolution shock capturing approach

- Two major algorithmic progresses in high order Godunov based MHD codes
  - Multidimensional integration algorithm
    - split vs. unsplit
    - Donor cell method
    - Corner transport upwind (CTU), Colella 1990

  - Divergence-free constraint on B
    - Hodge projection
      - Zachary et al, 1994; Crockett et al, 2005
    - Constrained transport (CT)
      - Evans & Hawley, 1988; Dai & Woodward, 1998; Ryu et al, 1998; Balsara & Spicer, 1999; Toth, 2000; Londrillo & Del Zanna, 2004; Ziegler, 2004; Gardiner & Stone, 2005; Fromang, 2006; Cunningham et al, 2007
    - Non-conservative formulation
      - Powell (et al), 1994; 1999; Falle et al, 1998; Dedner et al, 2002

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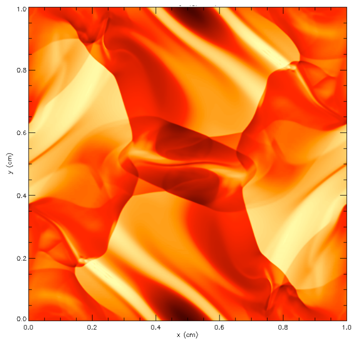
      - Zachary et al, 1994; Crockett et al, 2005
    - Constrained transport (CT)
      - Evans & Hawley, 1988; Dai & Woodward, 1998; Ryu et al, 1998; Balsara & Spicer, 1999; Toth, 2000; Londrillo & Del Zanna, 2004; Ziegler, 2004; Gardiner & Stone, 2005; Fromang, 2006; Cunningham et al, 2007
    - Non-conservative formulation
      - Powell (et al), 1994; 1999; Falle et al, 1998; Dedner et al, 2002

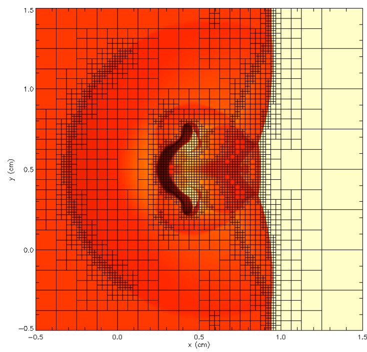
USM scheme

Applications

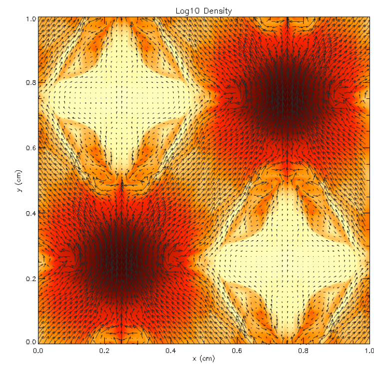
Orszag-Tang



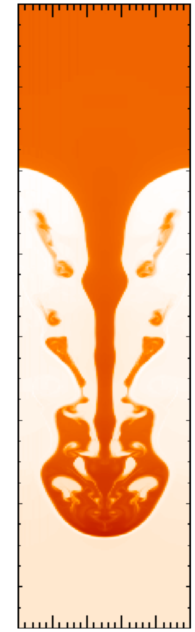
Shock-Cloud Interaction



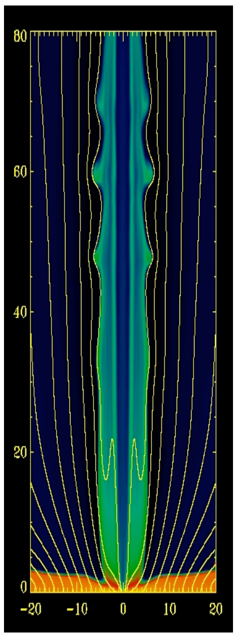
Self-Gravitating Plasma



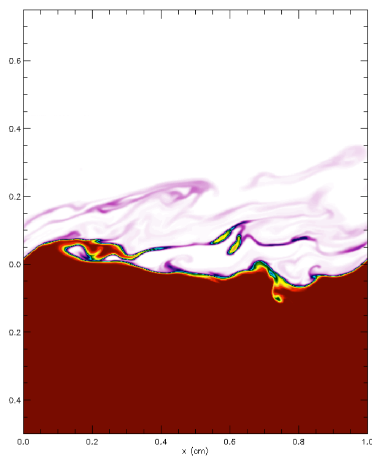
Magnetic RT



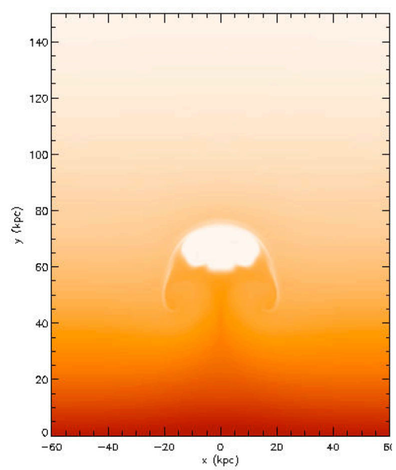
Jet Launching



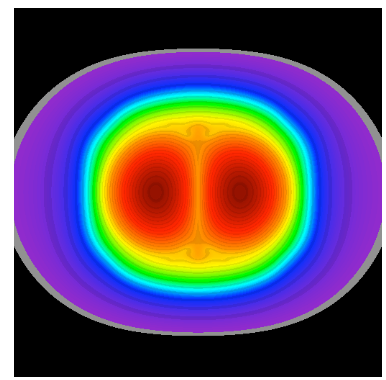
Surface Gravity Wave



Rising bubble



Magnetic reconnection



Beyond Plain MHD

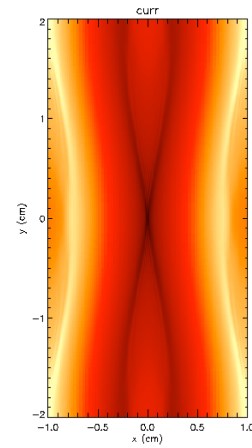
Plasma effects

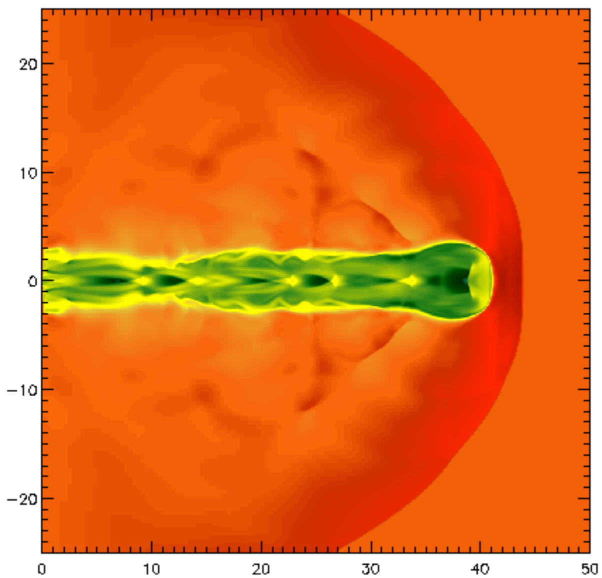
- Reduced 2D Hall (Grasso et al, 1999)
- Electron inertia and compressibility
- 3D Hall MHD and two-fluid MHD

$$\mathbf{E}+\mathbf{V}\times\mathbf{B}=\frac{1}{S}\mathbf{J}+\frac{d_e^2}{n}\frac{d\mathbf{J}}{dt}+\frac{d_i}{n}\left(\mathbf{J}\times\mathbf{B}-\nabla\cdot\vec{p}_e\right)$$

$$\begin{aligned}
\frac{\partial F}{\partial t}+[\phi,F] &= \rho_s^2[U,\psi]\\
\frac{\partial U}{\partial t}+[\phi,U] &= [J,\psi]\\
F &= \psi+d_e^2 J\\
J &= -\nabla^2\psi\\
U &= \nabla^2\phi
\end{aligned}$$

$$\begin{aligned}
\vec{B} &= B_0\hat{z}+\nabla\psi\times\hat{z}\\
\vec{v} &= \hat{z}\times\nabla\phi
\end{aligned}$$





Relativistic MHD

$$\frac{\partial\mathbf{W}}{\partial t}+(\nabla\cdot\mathbf{F})^{\mathrm{T}}=\mathbf{0}$$

$$\mathbf{W}=\begin{pmatrix}\Gamma_\rho\\ \Gamma^2\frac{e+p}{c^2}\mathbf{u}+\frac{1}{c^2}\mathbf{S}_{\mathrm{A}}\\ \mathbf{B}\\ \Gamma^2(e+p)-p-\Gamma\rho c^2+e_{\mathrm{A}}\end{pmatrix}\qquad \mathbf{F}=\begin{pmatrix}\Gamma\rho\mathbf{u}\\ \frac{\Gamma^2}{c^2}(e+p)\mathbf{u}\mathbf{u}+p\mathbf{I}+\mathbf{P}_{\mathrm{A}}\\ \mathbf{u}\mathbf{B}-\mathbf{B}\mathbf{u}\\ [\Gamma^2(e+p)-\Gamma\rho c^2]\mathbf{u}+\mathbf{S}_{\mathrm{A}}\end{pmatrix}^{\mathrm{T}}$$

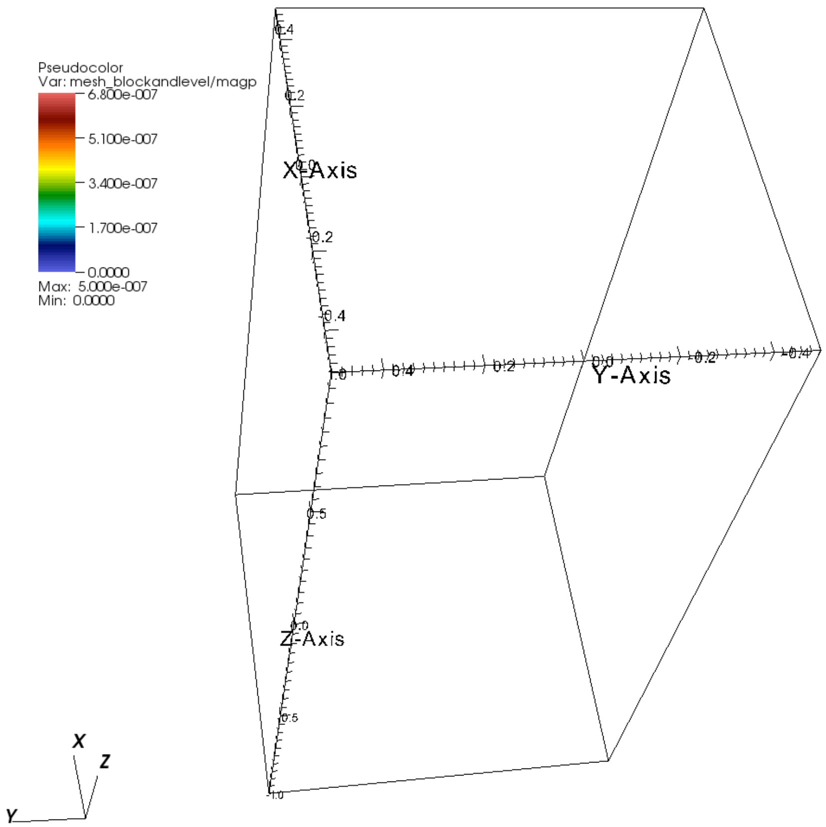
$$\Gamma=\frac{1}{\sqrt{1-\frac{u^2}{c^2}}},\qquad e_{\mathrm{A}}=\frac{1}{2\mu_0}\left(B^2+\frac{1}{c^2}E^2\right),$$

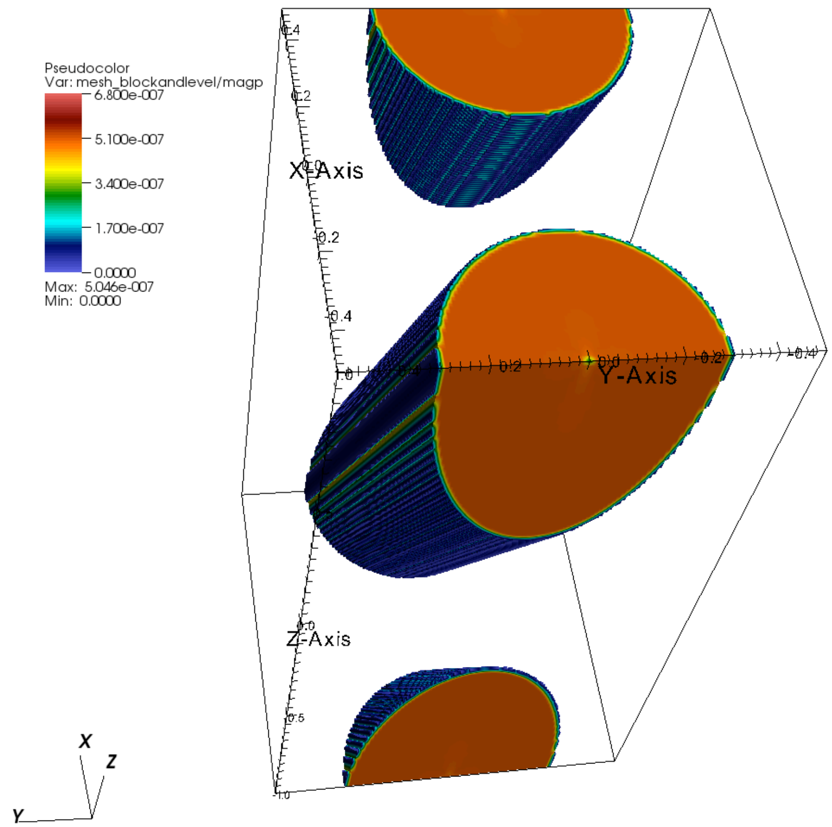
$$\mathbf{S}_{\mathrm{A}}=\frac{1}{\mu_0}(\mathbf{E}\times\mathbf{B}),\qquad \mathbf{P}_{\mathrm{A}}=e_{\mathrm{A}}\mathbf{I}-\frac{1}{\mu_0}\mathbf{B}\mathbf{B}-\frac{1}{\mu_0 c^2}\mathbf{E}\mathbf{E}.$$

Numerical Results on Benchmarked problems (1)

Field Loop advection in 3D

Gardiner & Stone (2008)

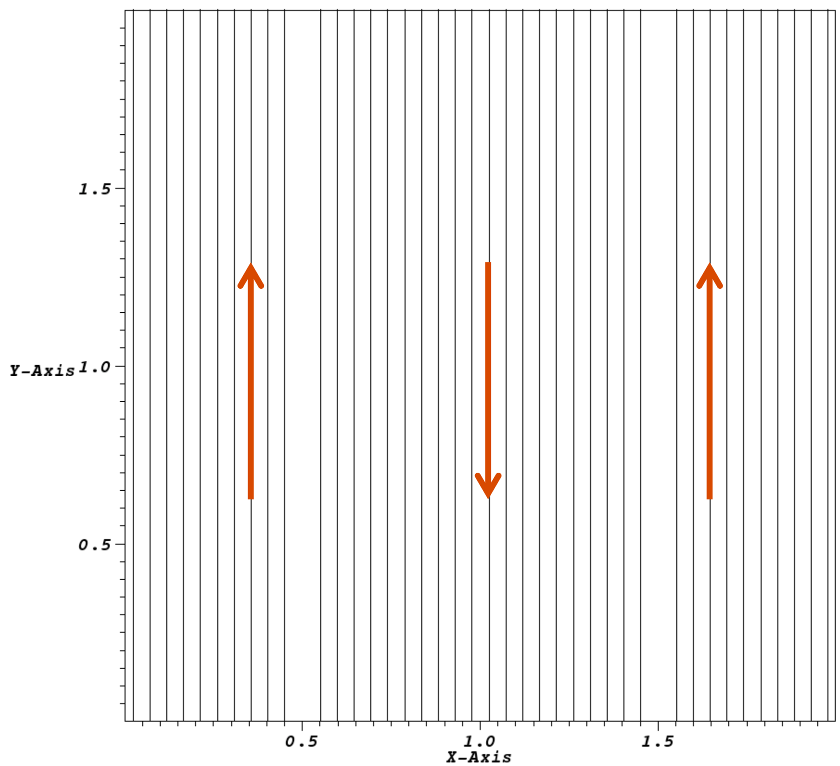




Numerical Results on Benchmarked problems (2)

Current Sheet Problem with different plasma beta values (magnetic field lines are shown)

$t = 0$



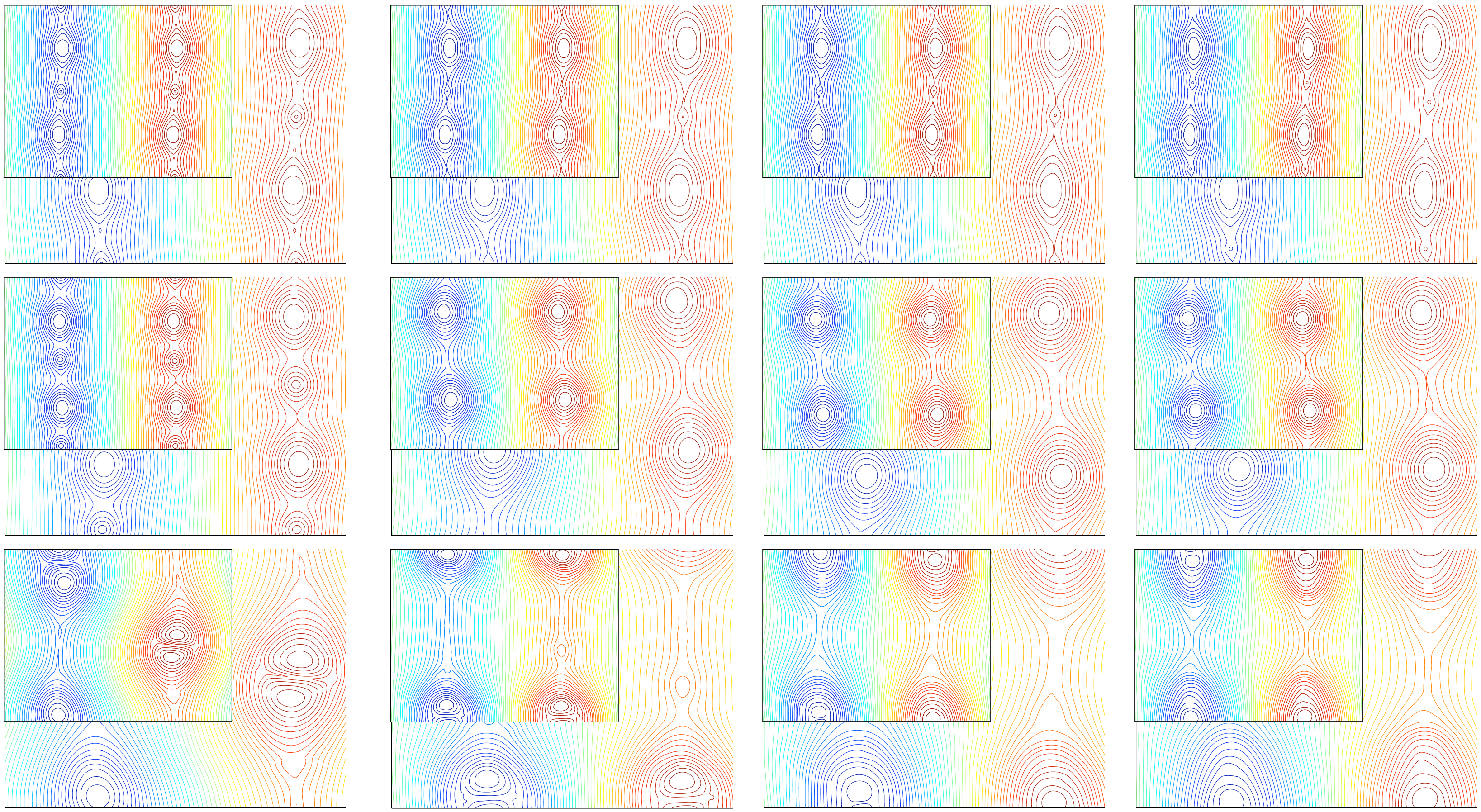
Numerical Results on Benchmarked problems (2)

Current Sheet Problem with different plasma beta values (magnetic field lines are shown)

$t = 2$

$t = 6$

$t = 10$



$\beta = 10^{-3}$ $\beta = 10^{-4}$ $\beta = 10^{-5}$ $\beta = 10^{-6}$

Other Applications